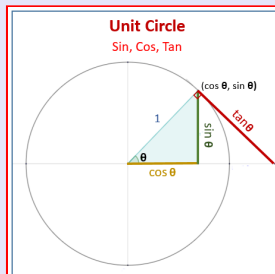


# Trigonometry

## Lecture 47



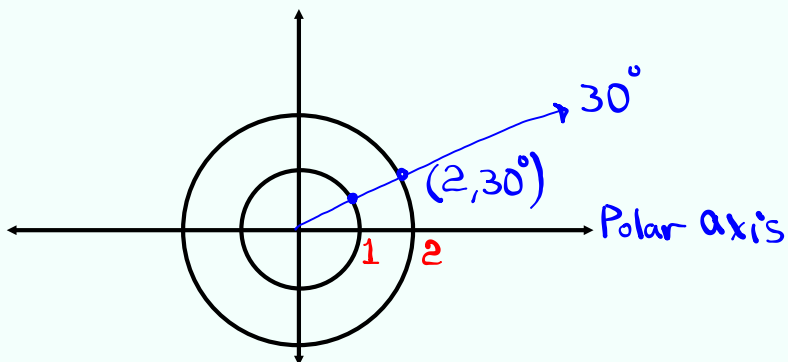
Polar Coordinate System

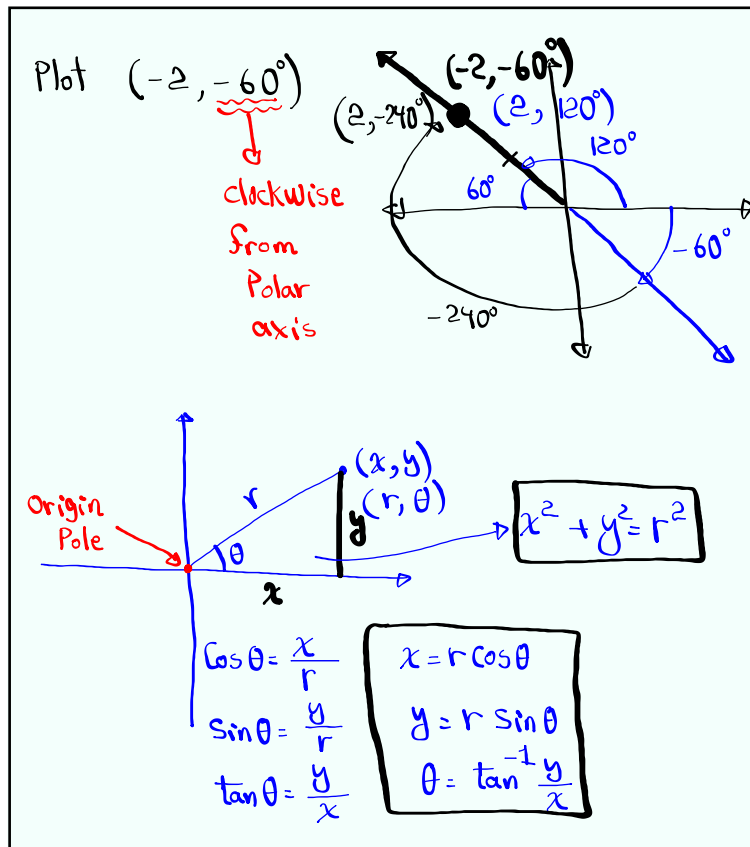
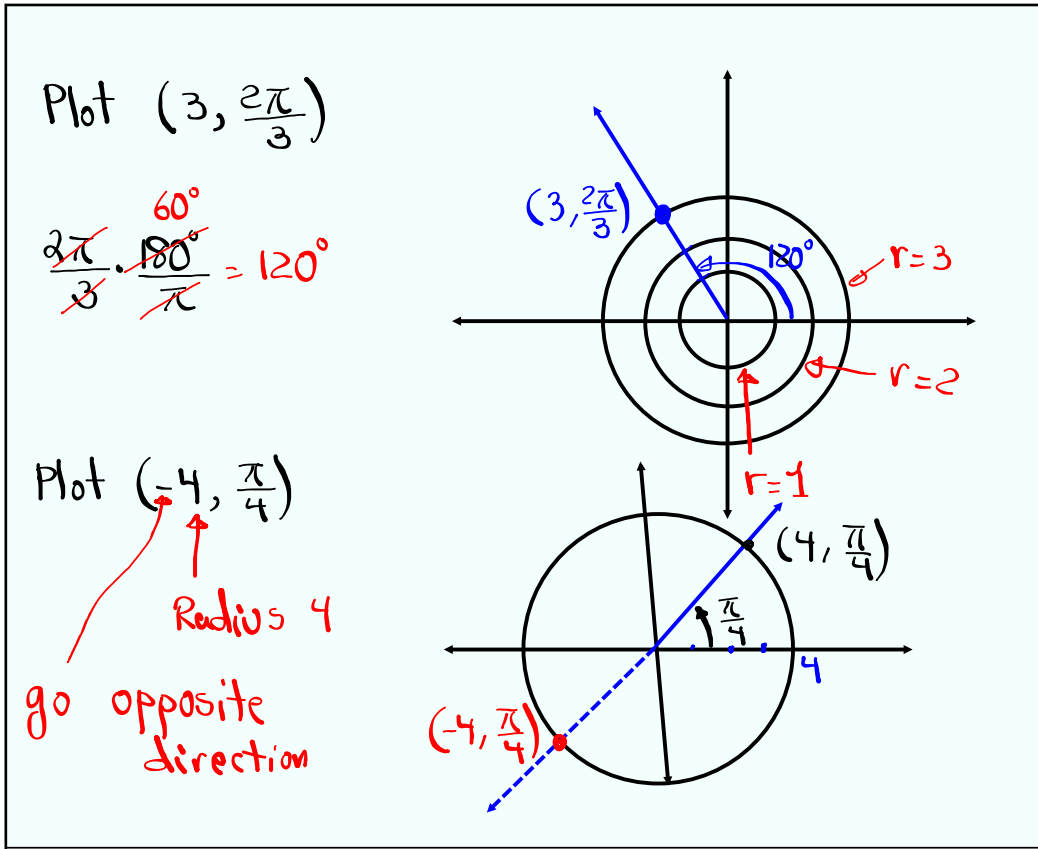
$$P(r, \theta) = P(x, y)$$

Rectangular  
Coordinate System

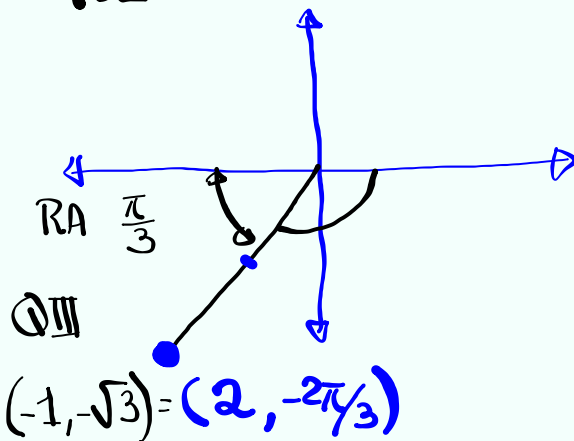
Plot  $(2, 30^\circ)$   
↑  
Radius

From polar axis ( $x$ -axis,  $x \geq 0$ )





Plot  $(2, -\frac{2\pi}{3})$  find its Cartesian Coordinates  
 $r=2$



Rectangular Coord.  
 $(x, y)$

$$\theta = -\frac{2\pi}{3}$$

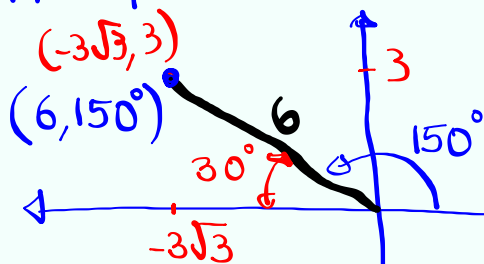
$$\theta = -120^\circ$$

↑  
 clockwise

$$x = r \cos \theta = 2 \cos\left(-\frac{2\pi}{3}\right) = 2 \cdot -\cos \frac{\pi}{3} = -2 \cdot \frac{1}{2} = -1$$

$$y = r \sin \theta = 2 \sin\left(-\frac{2\pi}{3}\right) = 2 \cdot -\sin \frac{\pi}{3} = -2 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3}$$

Plot  $(-3\sqrt{3}, 3)$  in rectangular, find  
 its polar form.



$$x = -3\sqrt{3}$$

$$y = 3$$

$$r^2 = x^2 + y^2$$

$$r^2 = (-3\sqrt{3})^2 + 3^2$$

$$= 9 \cdot 3 + 9$$

$$= 36 \quad \boxed{r=6}$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{3}{-3\sqrt{3}}$$

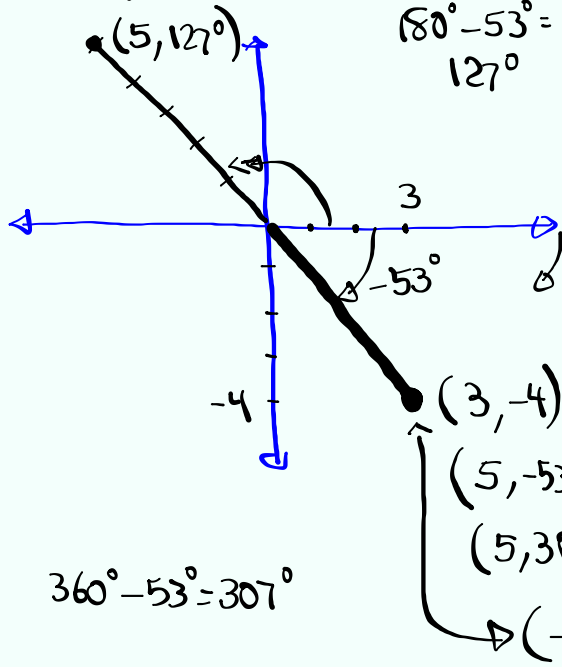
$$\tan \theta = -\frac{1}{\sqrt{3}}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$

$$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$$

$$\text{R.A. } 30^\circ$$

Plot  $(3, -4)$  in rectangular, find its polar form.



$$x=3 \quad y=-4$$

$$r^2 = x^2 + y^2$$

$$r^2 = 3^2 + (-4)^2 = 25$$

$$\boxed{r=5}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{-4}{3}\right) \approx \text{RA } 53^\circ$$

$$x^2 + y^2 = r^2$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

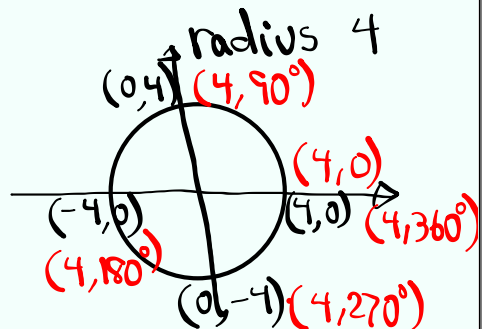
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Graph  $r^2 = 16$

$$x^2 + y^2 = 16$$

Circle center at  $(0,0)$

radius 4



Graph

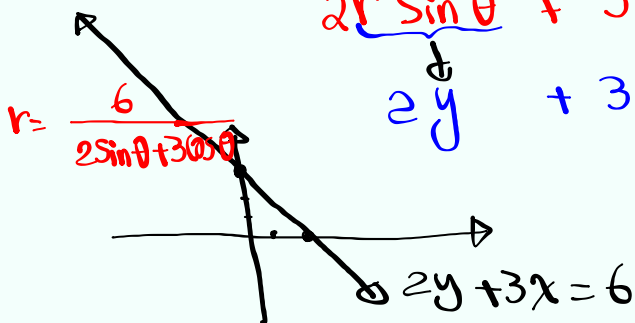
$$r = \frac{6}{2\sin\theta + 3\cos\theta}$$

Cross-Multiply

$$2r\sin\theta + 3r\cos\theta = 6$$

$$2y + 3x = 6$$

x	y
0	3
2	0



Graph

$$r = \frac{-10}{2\sin\theta - 5\cos\theta}$$

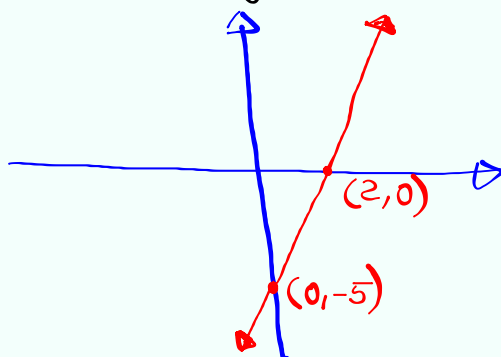
Hint:

Cross-Multiply

$$2r\sin\theta - 5r\cos\theta = -10$$

$$2y - 5x = -10$$

x	y
0	-5
2	0



Graph  $r = 4 \cos \theta$  Hint: Multiply  
 $r^2 = 4r \cos \theta$  both sides by  $r$

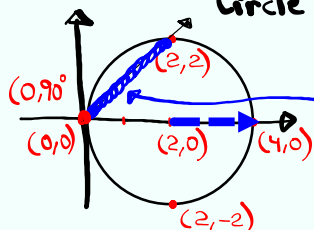
$$x^2 + y^2 = 4x$$

$$\underbrace{x^2 - 4x + 4} + y^2 = 0 + 4$$

$$(x-2)^2 + y^2 = 4$$

Circle center  $(2, 0)$

Radius 2



$$r = 4 \cos \theta$$

$$2 = 4 \cos \theta$$

$$\cos \theta = \frac{1}{2} \quad \theta = 60^\circ$$

$$\theta = 90^\circ \quad r = 4 \cos 90^\circ = 4 \cdot 0 = 0 \quad \boxed{r=0} \\ (0, 90^\circ)$$

Graph  $r = 8 \sin \theta$

Multiply by  $r$

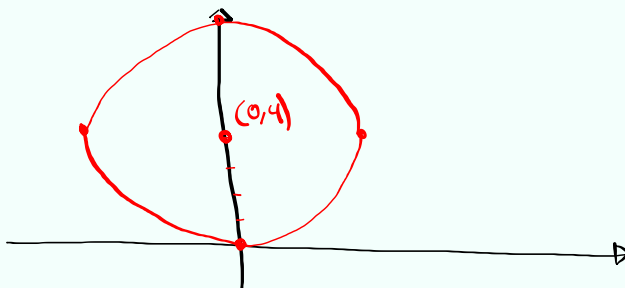
$$r^2 = 8r \sin \theta$$

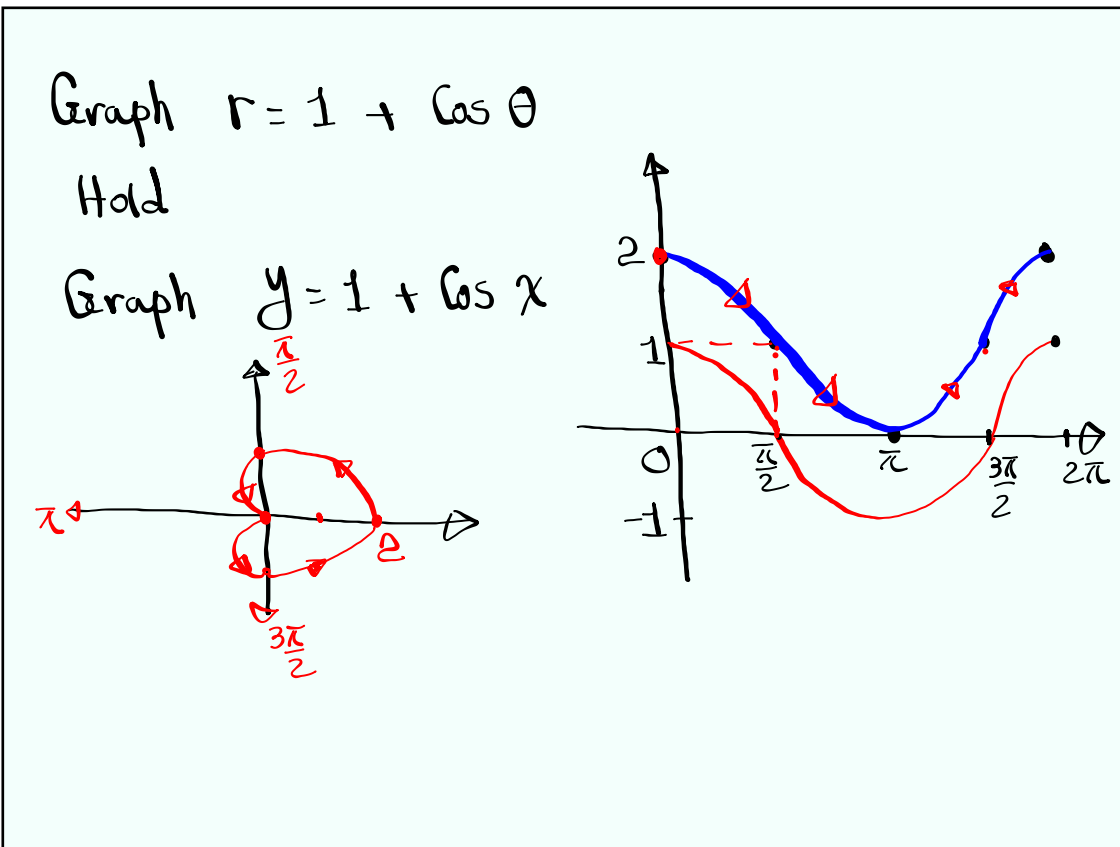
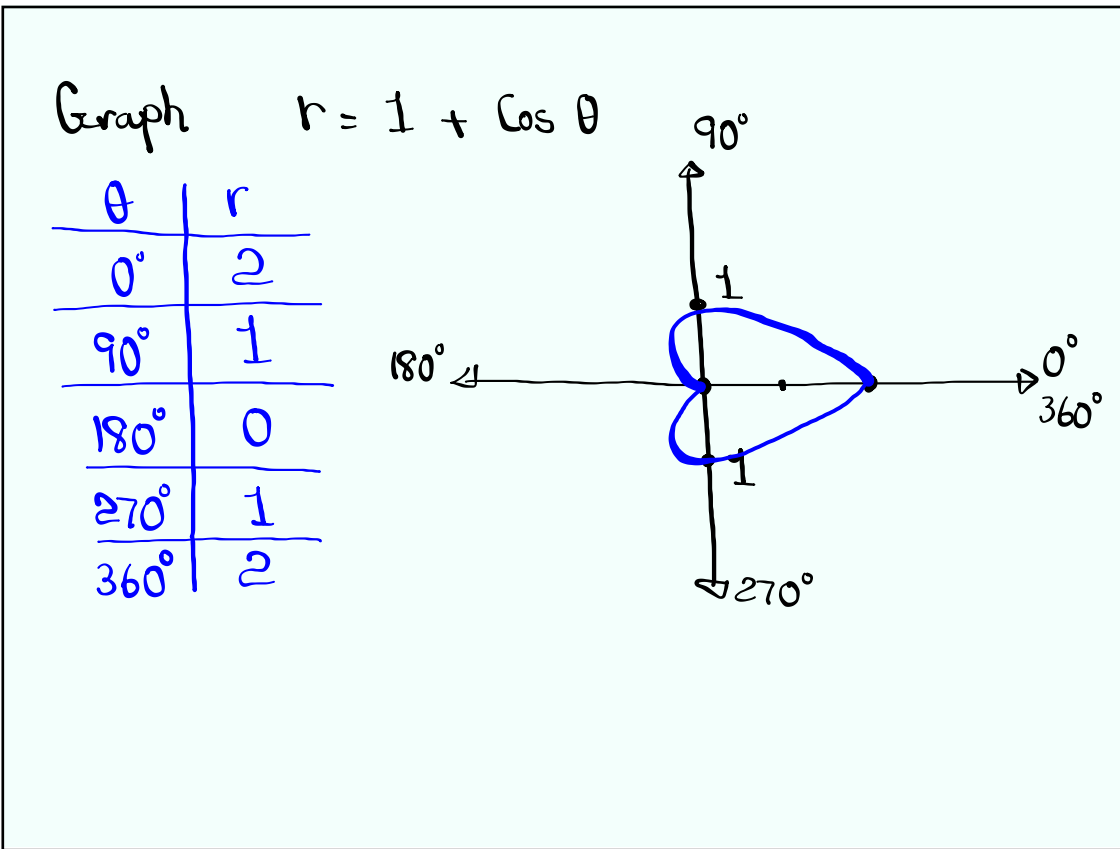
$$x^2 + y^2 = 8y$$

$$x^2 + y^2 - 8y + 16 = 0 + 16$$

$$x^2 + (y-4)^2 = 16$$

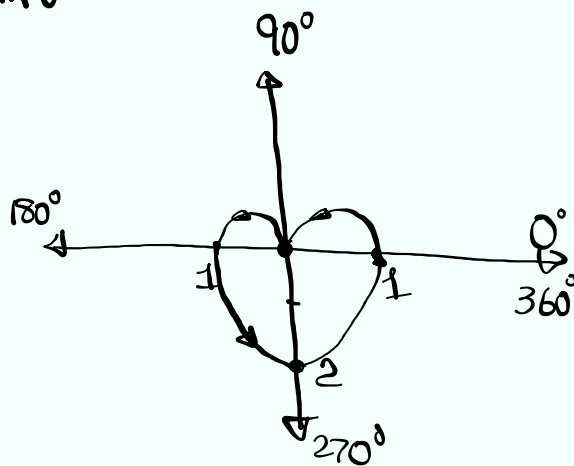
Center  $(0, 4)$  radius 4





Graph  $r = 1 - \sin \theta$

$\theta$	$r$
$0^\circ$	1
$90^\circ$	0
$180^\circ$	1
$270^\circ$	2
$360^\circ$	1



Graph  $r = 1 - \sin \theta$

Hold!

Graph  $y = 1 - \sin x$

